Chiral magnetic effect & anomalous transport from real-time lattice simulations

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CME in Heavy-Ion Collisions

Quantitative theoretical understanding of anomaly induced transport phenomena (CME,CMW,…) in heavy-ion collisions important experimental searches for these effects

Challenges:

Since life time of magnetic field is presumably very short (~0.1-1 fm/c) system is out-of-equilibrium during the time scales relevant for CME & Co.
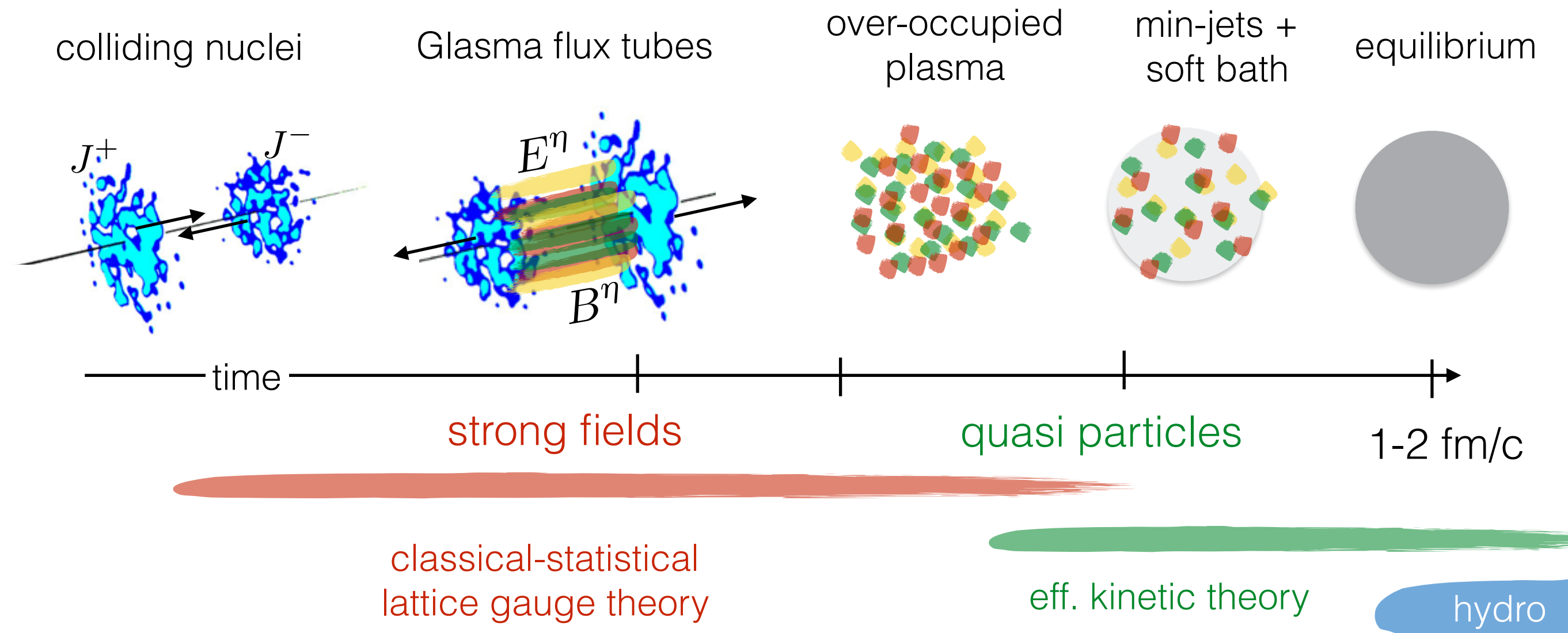
Need to understand non-equilibrium dynamics of axial and vector charges during the early-time pre-equilibrium phase

Existing theoretical approaches such as anomalous hydro or chiral kinetic theory effectively treat axial charge as a conserved quantity

In order to correctly describe generation of axial charge imbalance (e.g. due to sphalerons) field theoretical description is required

-> Develop field theoretical approach to describe early time dynamics and possibly devise improved macroscopic description of anomalous transport

Early-time dynamics of HIC

colliding nuclei · Glasma flux tubes · over-occupied plasma · min-jets + soft bath · equilibrium

J^+ J^- E^η B^η

time

strong fields · quasi particles · 1-2 fm/c

classical-statistical lattice gauge theory

eff. kinetic theory

hydro

Early time dynamics described in terms of classical field dynamics amenable to non-perturbative real-time lattice simulations

-> Include dynamical fermions to study anomalous transport

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Simulation technique

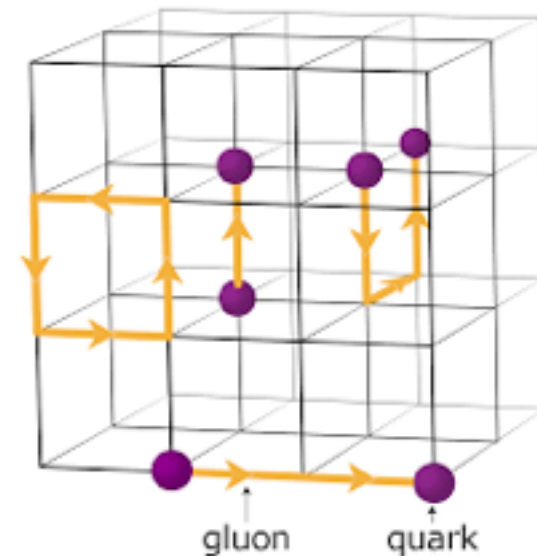**_Classical-statistical lattice simulation with dynamical fermions_**

(Aarts, Smit; Berges, Hebenstreit,Kasper, Mueller; Tranberg, Saffin; …)

- Discretize theory on 3D spatial lattice using the Hamiltonian lattice formalism

- Solve operator Dirac equation in the presence of SU(N) and U(1) gauge fields

$$i\gamma^0 \partial_t \hat{\psi} = (-i\slashed{D}^s_W + m)\hat{\psi}$$

gluon   quark

- Compute expectation values of vector and axial currents to study anomalous transport processes

$$j_v^\mu(x) = \langle \hat{\bar{\psi}}(x)\gamma^\mu\hat{\psi}(x) \rangle \quad j_a^\mu(x) = \langle \hat{\bar{\psi}}(x)\gamma^\mu\gamma^5\hat{\psi}(x) \rangle$$

Dynamical fermions

Solving the operator Dirac equation can be achieved by expanding the fermion field in operator basis at initial time

$$\hat{\psi}(x,t) = \sum_{p,\lambda} \hat{b}_{p,\lambda}(t=0)\phi_u^{p,\lambda}(x,t) + \hat{d}_{p,\lambda}^\dagger(t=0)\phi_v^{p,\lambda}(x,t)$$

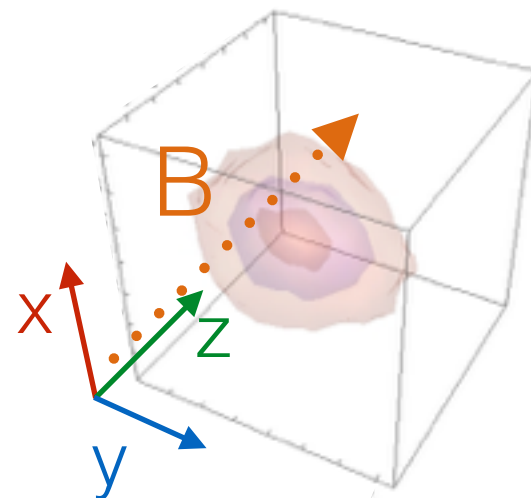and solving the Dirac equation for evolution of $4N_cN^3$ wave-functions

Not clear to what extent stochastic estimators are useful to reduce problem size
Computationally extremely demanding (~TB memory, ~M CPU hours)

So far first results on small lattices 24 x 24 x 64 in a clean theoretical setup

SU(N): Single sphaleron transition
U(1):   constant magnetic field

Back-reaction of fermions on
gauge field evolution not considered

B

x
z
y

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Axial anomaly in real-time

Definition of chiral properties (axial charge) of fermions on the lattice generally a tricky issue

Naive fermion discretization: Cancellation of axial anomaly due to Fermion doublers

$$\partial_\mu j_5^\mu(x) = 2m < \bar{\psi}(x) i\gamma_5 \psi(x) >$$

Exploit knowledge from Euclidean lattice simulations

Wilson fermions: Explicit symmetry breaking term added to the Hamiltonian to decouple doublers (c.f. Aarts,Smit)

$$\partial_\mu j_5^\mu(x) = 2m < \bar{\psi}(x) i\gamma_5 \psi(x) > + r_W < W(x) > \xrightarrow{\text{cont. limit}} -\frac{g^2}{8\pi^2} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu}$$

Overlap fermions: Non-local derivative operator with chiral properties on the lattice

Axial anomaly in real-time
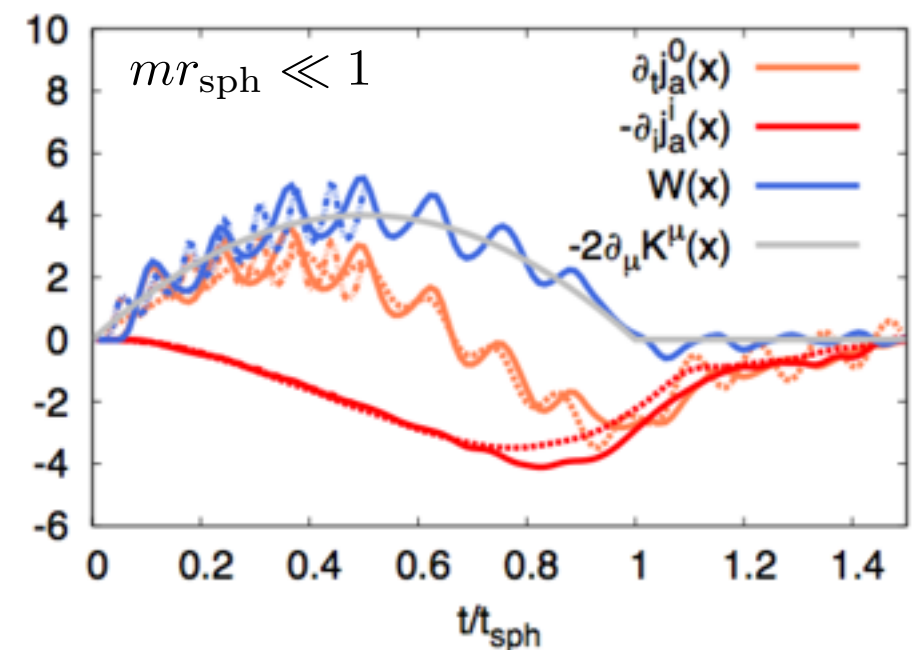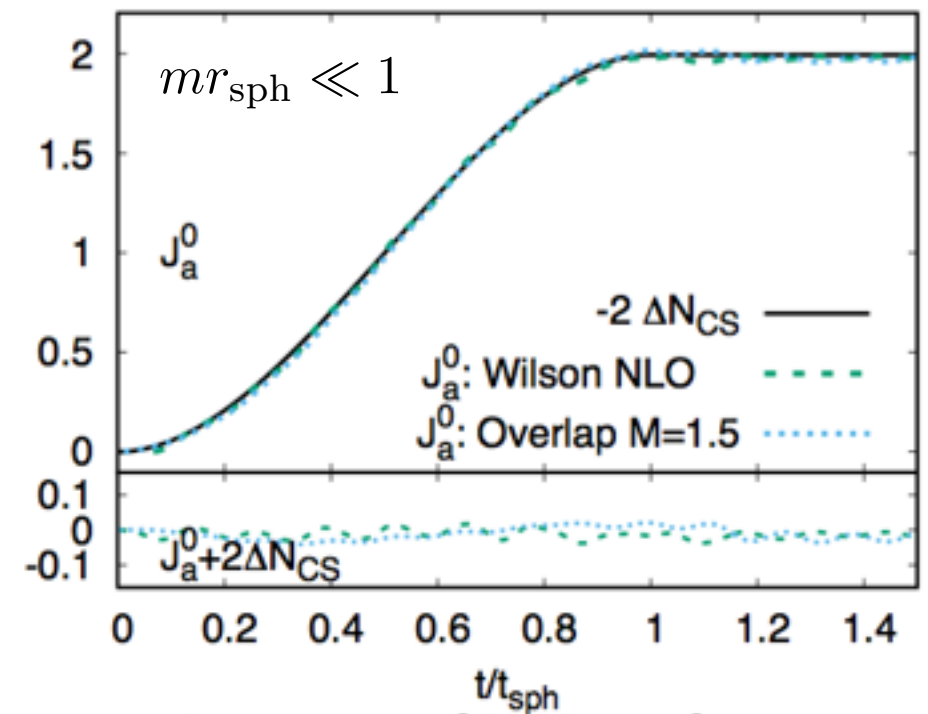
Non-trivial cross check of axial charge production (B=0)

Over the coarse of the sphaleron transition Chern-Simons number

$$\Delta N_{CS} = \frac{g^2}{8\pi^2} \int d^4x \ \vec{E}_a \vec{B}_a$$

changes by an integer amount leading to an imbalance of axial charge

$$\Delta J_5^0 = -2\Delta N_{CS} + 2m_f \int d^4x \langle \bar{\psi} i \gamma_5 \psi \rangle$$

Excellent agreement for (almost) massless fermions from simulations with improved Wilson fermions and Overlap fermions

(Mace,Mueller,SS, Sharma, 1612.02477)

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CME Dynamics

Axial charge j_5^0 Vector current j_V^z Vector charge j_V^0

x

z

y

Sphaleron transition induces local imbalance of axial charge density

Non-zero magnetic field B_z leads to vector current j_V^z in z-direction
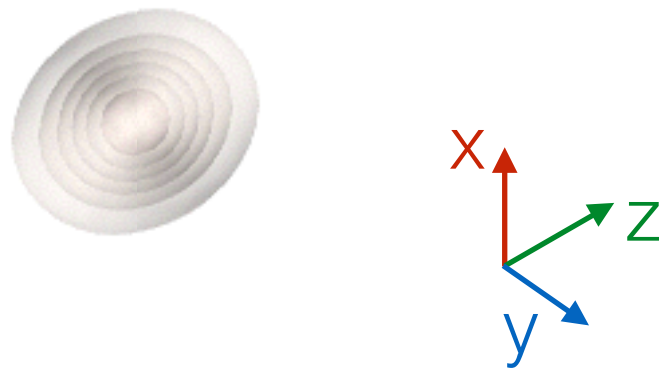
Vector current j_V^z leads to separation of electric charges j_V^0 along the z-direction

(N.Mueller,SS, S. Sharma PRL 117 (2016) no.14, 142301)
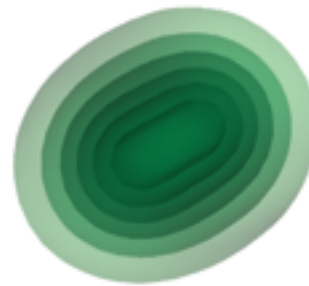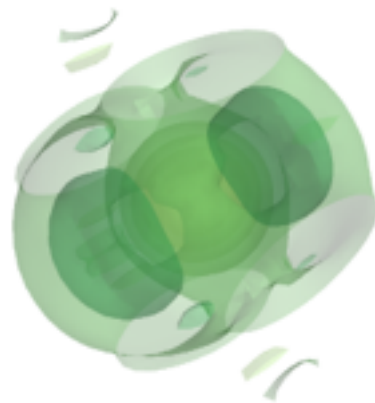
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CME Dynamics

Axial charge j_5^0 Vector current j_V^z Vector charge j_V^0

Sphaleron transition induces local imbalance of axial charge density

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CME Dynamics

Axial charge j_5^0

Vector current j_V^z

Vector charge j_V^0

x
z
y

Sphaleron transition induces local imbalance of axial charge density

Non-zero magnetic field B_z leads to vector current j_V^z in z-direction

Vector current j_V^z leads to separation of electric charges j_V^0 along the z-direction

CMW Dynamics

Vector charge imbalance j_V^0 generates an axial current j_5^z
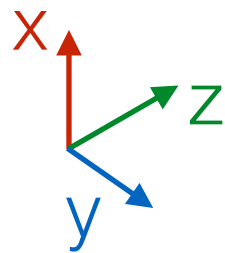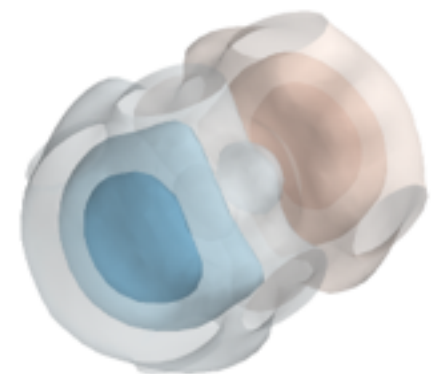so that axial charge also flows along the B-field direction

Axial charge j_5^0 Vector current j_V^z Vector charge j_V^0

Emergence of a Chiral Magnetic Shock-wave of vector charge and
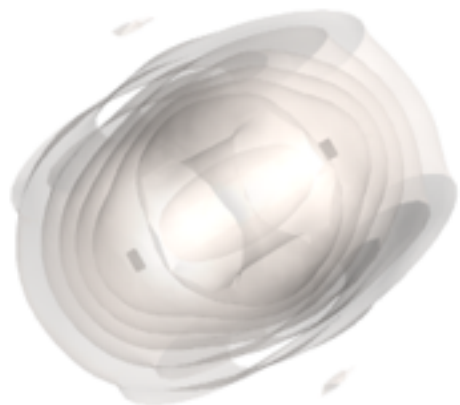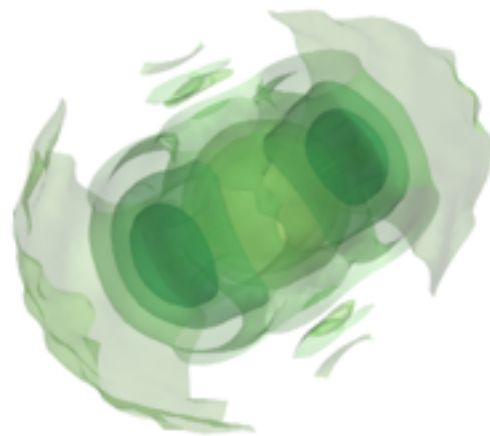axial charge propagating along B-field direction

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CMW Dynamics

Vector charge imbalance j_V^0 generates an axial current j_5^z
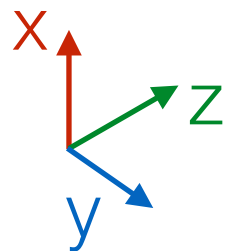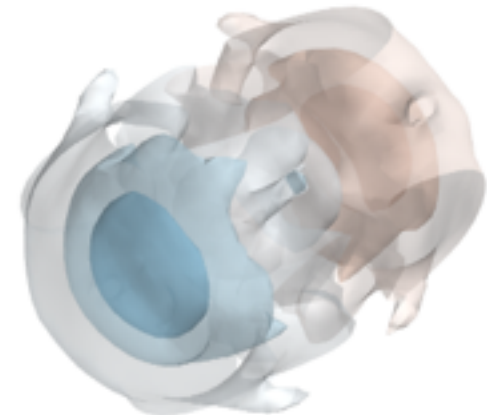so that axial charge also flows along the B-field direction

Axial charge j_5^0 Vector current j_V^z Vector charge j_V^0

Emergence of a Chiral Magnetic Shock-wave of vector charge and
axial charge propagating along B-field direction

(N.Mueller,SS, S. Sharma PRL 117 (2016) no.14, 142301)

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CMW Dynamics

Vector charge imbalance j_V^0 generates an axial current j_5^z
so that axial charge also flows along the B-field direction

Axial charge j_5^0 Vector current j_V^z Vector charge j_V^0

Emergence of a Chiral Magnetic Shock-wave of vector charge and
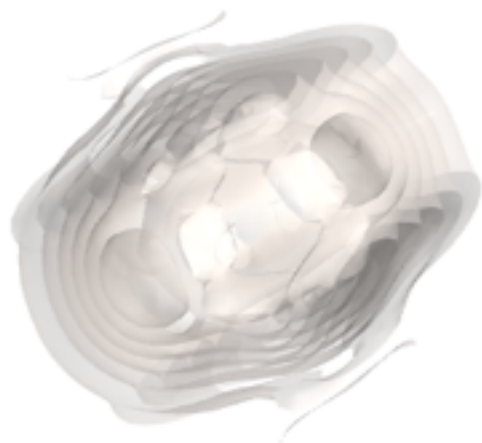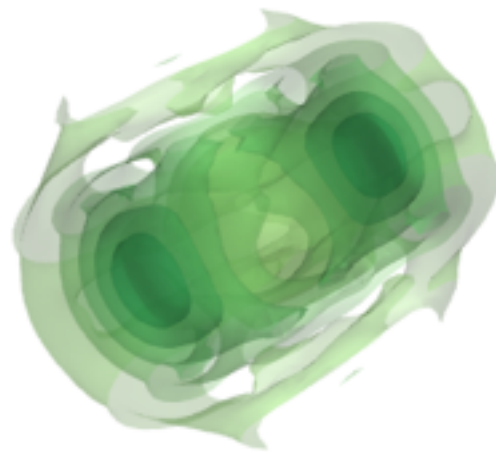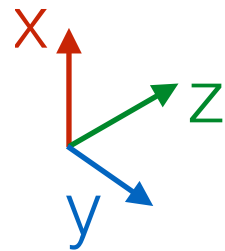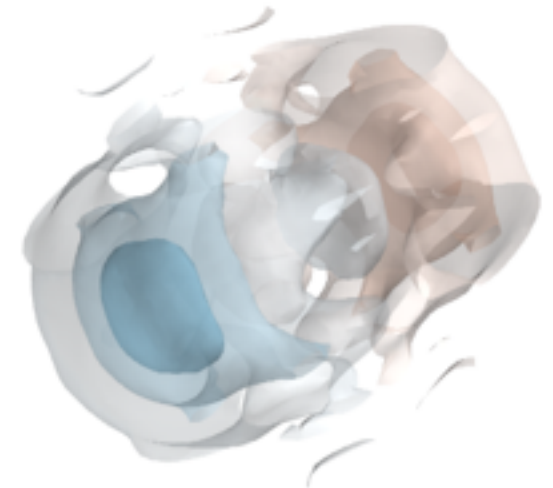axial charge propagating along B-field direction

Non-equilibrium dynamics of vector and axial charges

axial charge j_a^0

vector charge j_v^0

time

Clear separation of electric charge j_V^0 along the B-field direction

First time anomalous transport phenomena have been confirmed from non-perturbative real-time simulations

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Non-equilibrium dynamics of vector and axial charges

Comparison with anomalous hydro (light quarks $mr_{\rm sph} \ll 1$)

$$\partial_\mu j_a^\mu = S(x) , \qquad \partial_\mu j_v^\mu = 0$$

$$j_{v/a}^\mu = n_{v/a} u^\mu + \sigma_{v/a}^B B^\mu$$
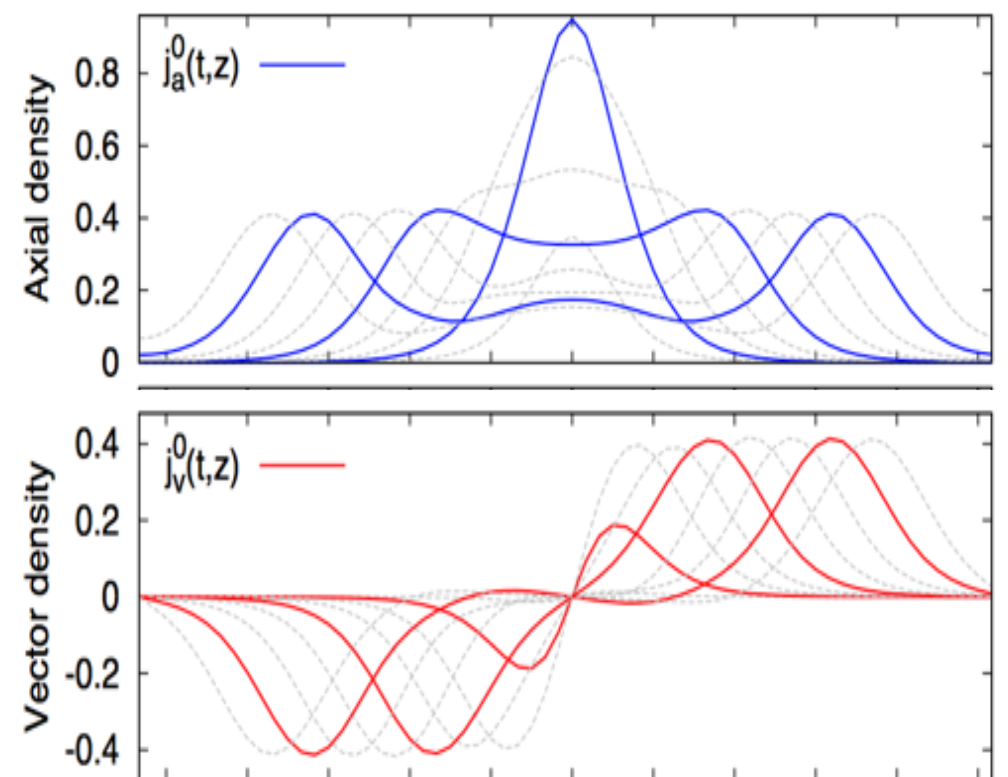
Strong field limit ($B \gg r_{\rm sph}^{-2}, m^2$)

$$\partial_t \begin{pmatrix} j_v^0(t,z) \\ j_a^0(t,z) \end{pmatrix} = -\partial_z \begin{pmatrix} j_a^0(t,z) \\ j_v^0(t,z) \end{pmatrix} + \begin{pmatrix} 0 \\ S(t,z) \end{pmatrix}$$

Simulation results for light quarks

Chiral magnetic shock-wave

$$j_{v/a}^0(t > t_{\rm sph}, z) = \frac{1}{2} \int_0^{t_{\rm sph}} dt' \Big[S(t', z - c(t - t')) \mp S(t', z + c(t - t')) \Big]$$

-> Evolution for light quarks and strong magnetic fields
well described by anomalous hydrodynamics at late times

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Validity of constitutive relations

CSE

CME

qB = 0.8 r_{sph}^{-2}
qB = 1.6 r_{sph}^{-2}
qB = 3.5 r_{sph}^{-2}
qB = 7 r_{sph}^{-2}

(Mace,Mueller,SS, Sharma, 1612.02477)

Verify ratios vector/axial currents and axial/vector charge

$$C_{\mathrm{CME}}(t) = \frac{\Delta J_v^z(t)}{\Delta J_a^0(t)}, \qquad C_{\mathrm{CSE}}(t) = \frac{\Delta J_a^z(t)}{\Delta J_v^0(t)}.$$
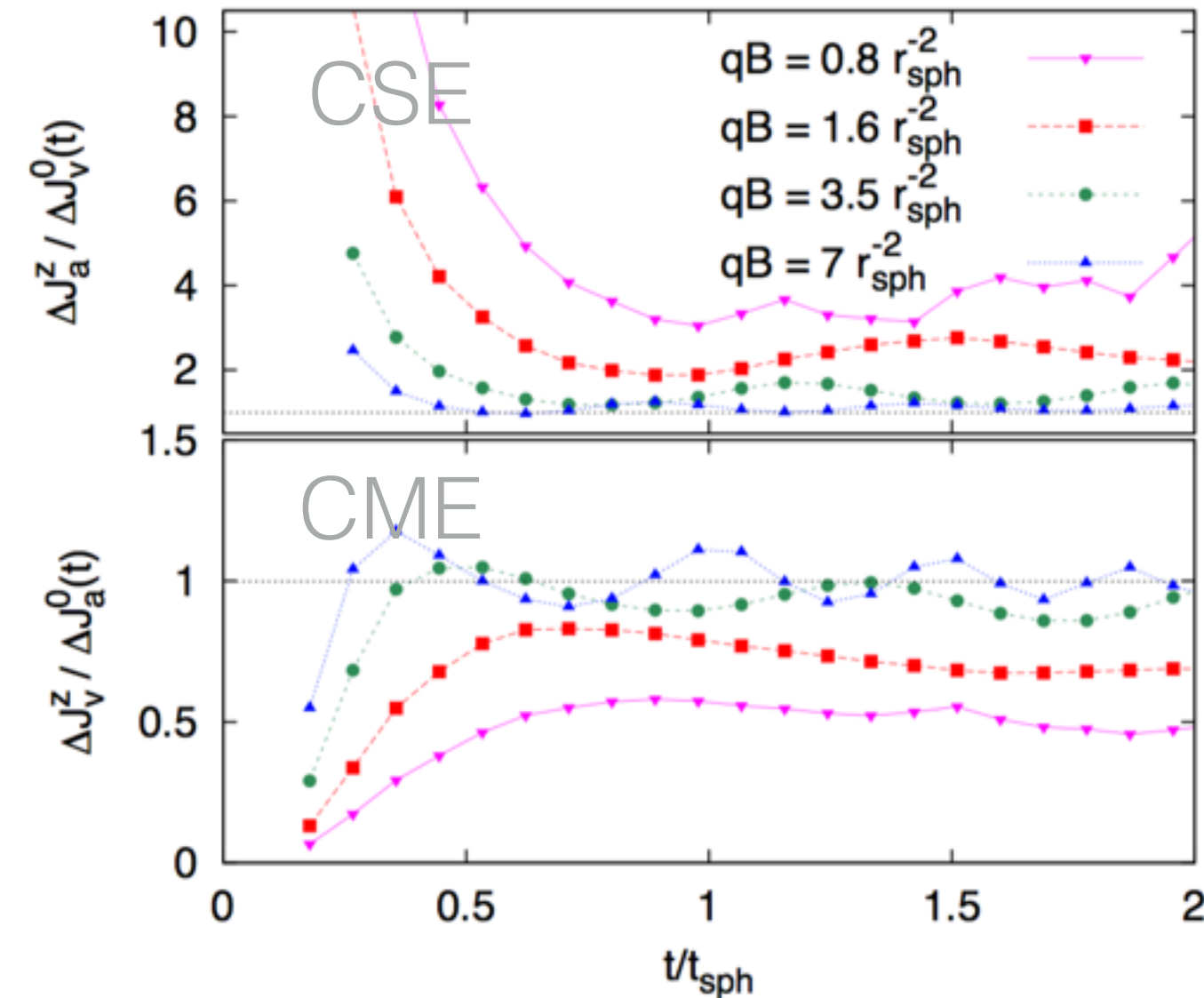
In the strong field limit related to thermodynamic constitutive relations

$$C_{CME} = 1, \qquad C_{CSE} = 1.$$

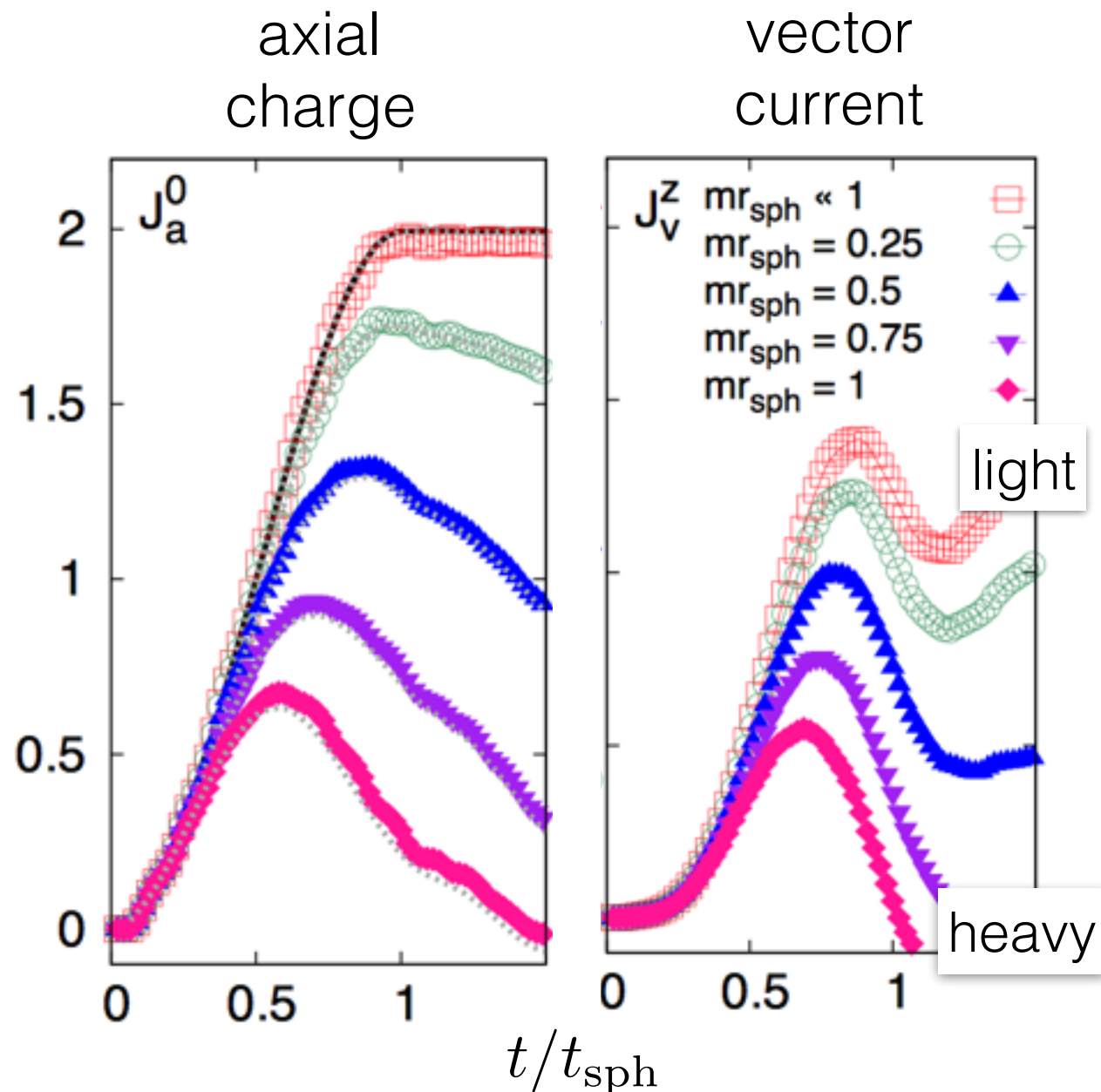equal to time independent constants.

Simulation results indicate approach towards constant value with a finite relaxation time

Since lifetime of magnetic field in HIC is short this effect should also be incorporated in phenomenological approaches

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Quark mass dependence

axial charge

vector current

Explicit violation of axial charge conservation for finite quark mass

$$\partial_\mu j_a^\mu(x) = 2m\langle\hat{\bar{\psi}}(x)i\gamma_5\hat{\psi}(x)\rangle + S(x)$$
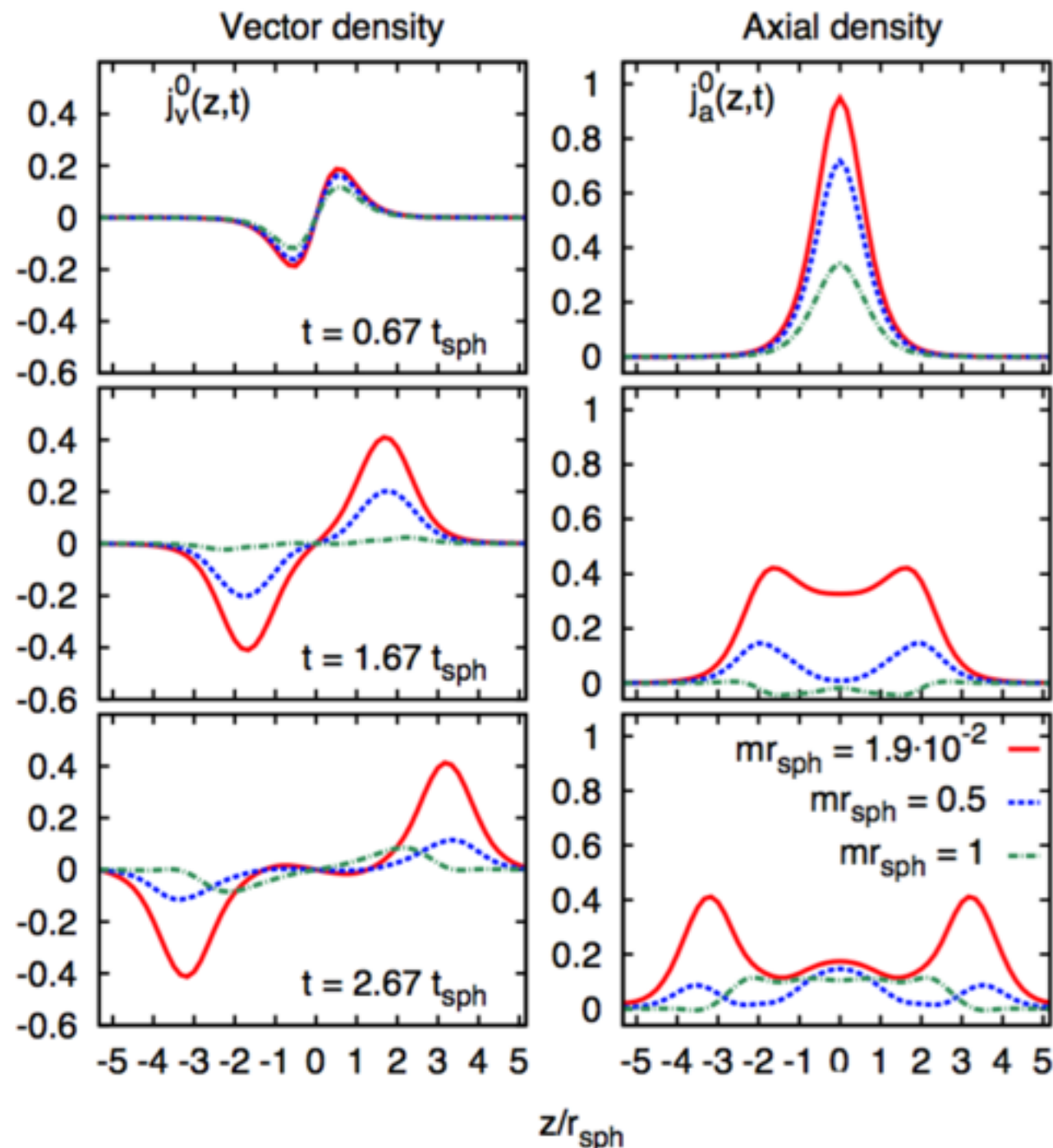
leads to damping of axial charge

Since chiral magnetic effect current is proportional to axial charge density it will also be reduced

$$\vec{j}_v \propto j_a^0\vec{B}$$

(N.Mueller,SS, S. Sharma PRL 117 (2016) no.14, 142301)
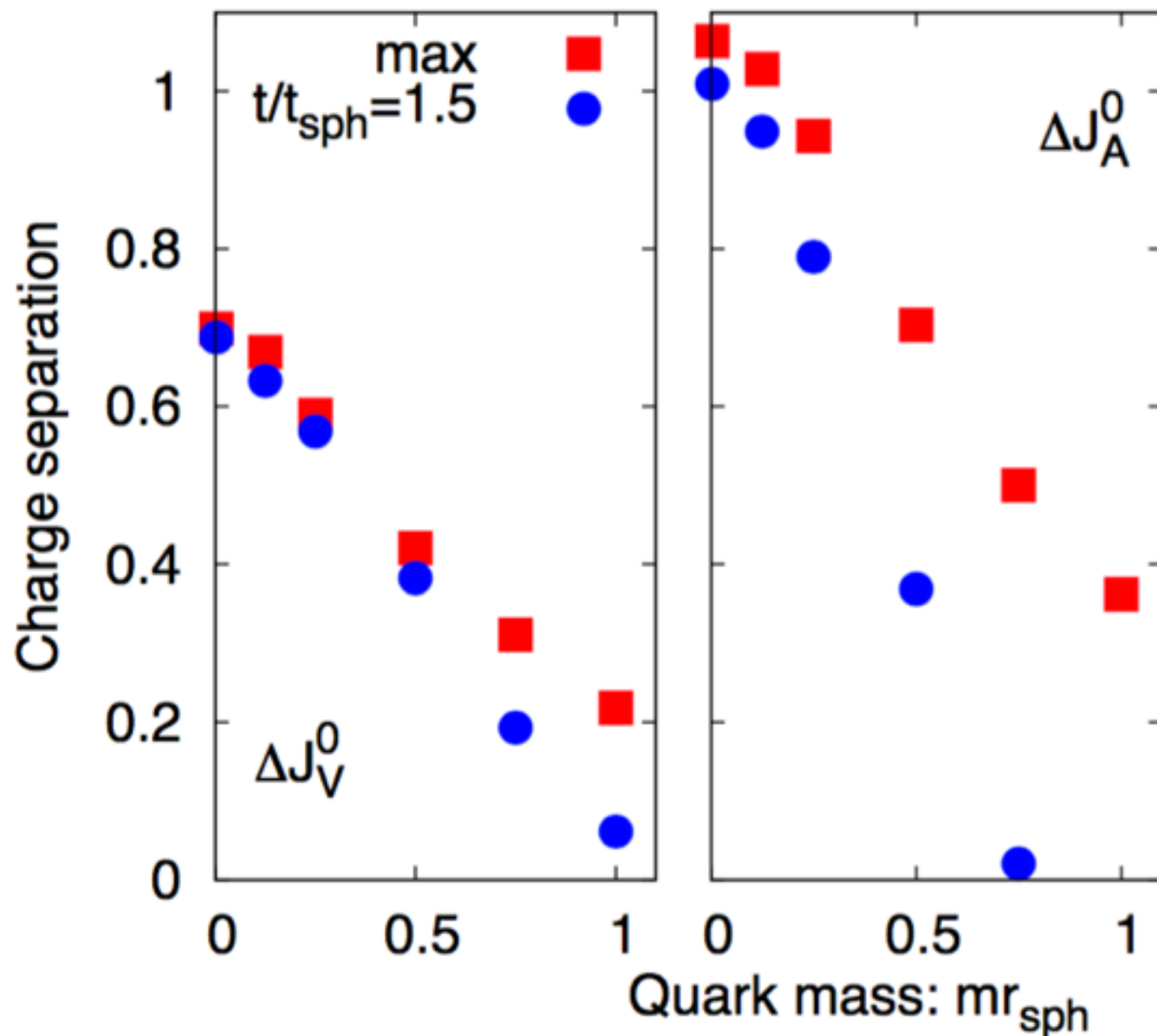
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Quark mass dependence

Vector density   **Axial density**

$j_v^0(z,t)$ $j_a^0(z,t)$

$t = 0.67\ t_{sph}$

$t = 1.67\ t_{sph}$

$t = 2.67\ t_{sph}$

$mr_{sph} = 1.9\cdot10^{-2}$
$mr_{sph} = 0.5$
$mr_{sph} = 1$

z/r_{sph}

Light quarks ($\mathbf{mt_{sph} \ll 1}$ *)*

Chiral magnetic wave leads to non-dissipative transport of axial and vector charges

Heavy quarks ($\mathbf{mt_{sph} \sim 1}$ *)*

Dissipation of axial charge leads to significant reduction of charge separation

(Mace,Mueller,SS, Sharma, 1612.02477)

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Quark mass dependence

Significant reduction of the charge separation signal by factor ~5 already for moderate quark masses

Expect backreaction (not included so far) to suppress the signal even further

Phenomenological consequences

Unlikely that strange quarks participate in CME

Desirable to include dissipative effects in macroscopic description

(Mace,Mueller,SS, Sharma, 1612.02477)

Conclusions & Outlook

Development of first-principle techniques to study dynamics of
vector and axial charges out-of-equilibrium

Successful microscopic description of CME & CMW

Observed importance of finite relaxation time and dissipative effects

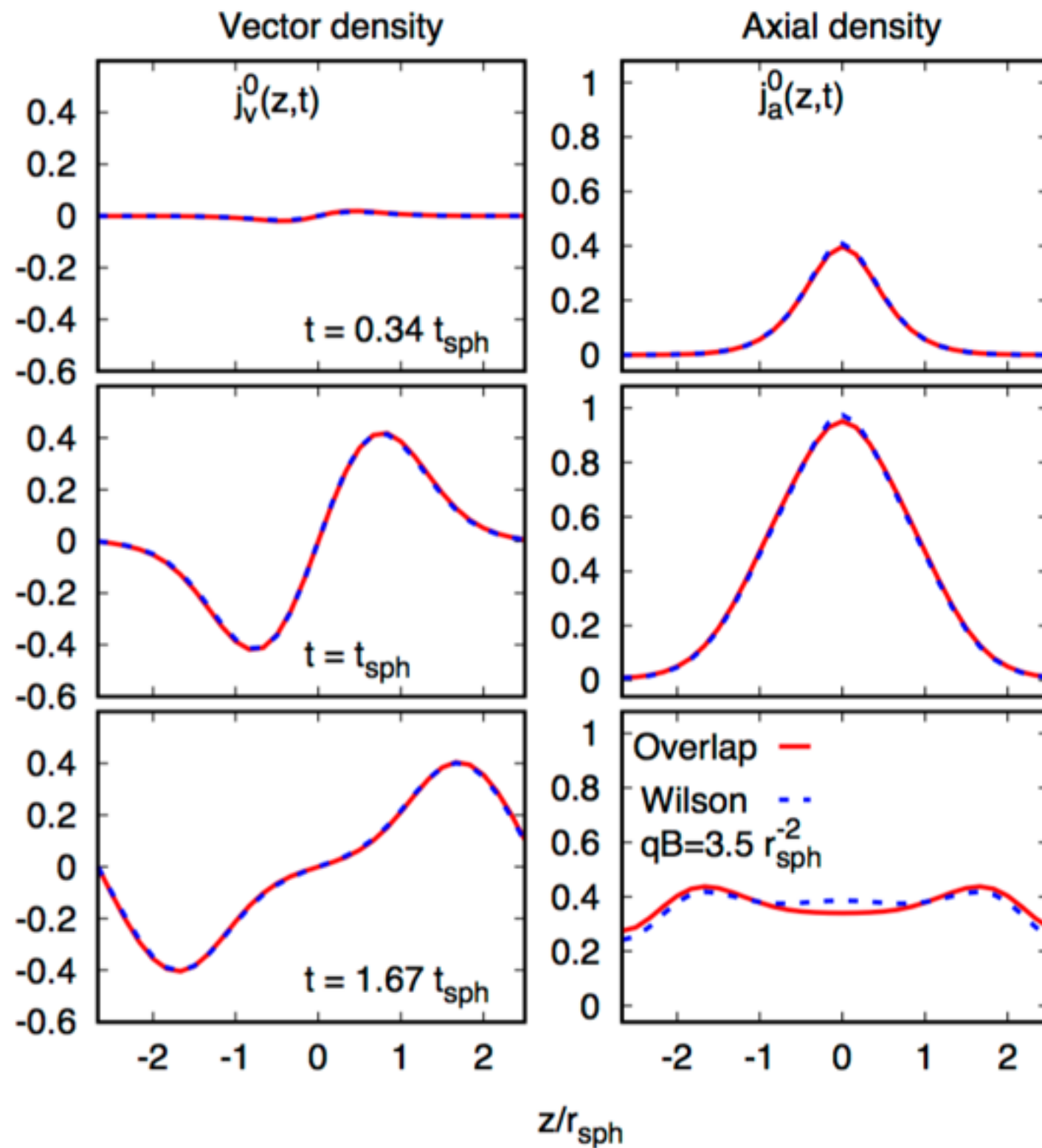Should be included in macroscopic descriptions

Next step is to include back-reaction of fermions perform
simulations for a realistic heavy-ion environment

- Quark production & electro-magnetic response
- Chiral magnetic effect & anomalous transport in HIC

Several technical developments in progress to
reduce numerical complexity / extend lattice size

Backup

Comparison of Wilson & Overlap

Magnetic field dependence